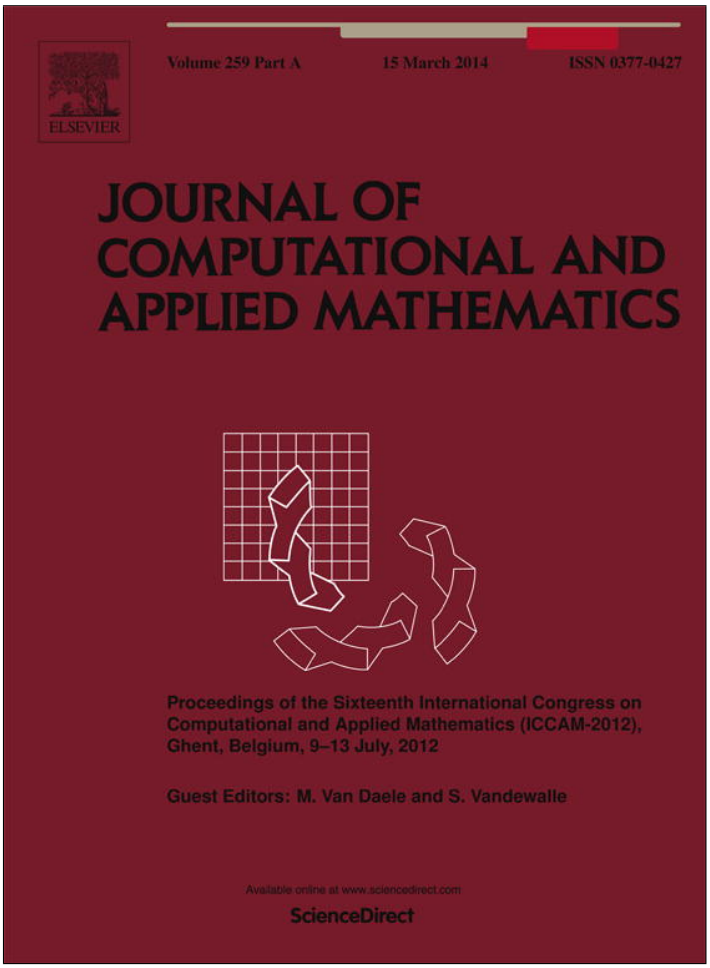

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Numerical approximation of a two-dimensional parabolic time-dependent problem containing a delta function☆

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ABSTRACT

The convergence of a difference scheme for a two-dimensional initial-boundary value problem for the heat equation with concentrated capacity and time-dependent coefficients of the space derivatives is considered. An estimate of the rate of convergence in a special discrete $\widetilde{W}_2^{2,1}$ Sobolev norm, compatible with the smoothness of the coefficients and the solution, is proved.

1. Introduction

The finite-difference method is one of the basic tools for the numerical solution of partial differential equations. In the case of problems with discontinuous coefficients and concentrated factors (Dirac delta functions, free boundaries, etc.), the solution has weak global regularity, and it is impossible to establish convergence of finite-difference schemes using the classical Taylor series expansion. Often, the Bramble–Hilbert lemma takes the role of the Taylor formula for functions from the Sobolev spaces [1–3].

Following Lazarov et al. [3], a convergence rate estimate of the form

$$\|u - v\|_{W_{2,h}^k} \le Ch^{s-k}\|u\|_{W_2^s}, \quad s > k,$$

is called *compatible* with the smoothness (regularity) of the solution u of the boundary value problem. Here, v is the solution of the discrete problem, h is the spatial mesh step, W_2^s and $W_{2,h}^k$ are Sobolev spaces of functions with continuous and discrete argument, respectively, and C is a constant which does not depend on u and h. For parabolic problems, typical estimates are of the form

$$\|u - v\|_{W_{2,h\tau}^{k,k/2}} \le C(h + \sqrt{\tau})^{s-k}\|u\|_{W_2^{s,s/2}}, \quad s > k,$$

where τ is the time step. In the case of equations with variable coefficients, the constant C in the error bounds depends on the norms of the coefficients (see, for example, [2,4,5]).

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