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Numerical approximation of a two-dimensional parabolic time-dependent problem containing a delta function[☆]



Dejan R. Bojović^{a,*}, Bratislav V. Sredojević^a, Boško S. Jovanović^b

^a University of Kragujevac, Faculty of Science, R. Domanovića 12, 34000 Kragujevac, Serbia

^b University of Belgrade, Faculty of Mathematics, Studentski trg 16, 11000 Belgrade, Serbia

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ABSTRACT

The convergence of a difference scheme for a two-dimensional initial-boundary value problem for the heat equation with concentrated capacity and time-dependent coefficients of the space derivatives is considered. An estimate of the rate of convergence in a special discrete $\tilde{W}_2^{2,1}$ Sobolev norm, compatible with the smoothness of the coefficients and the solution, is proved.

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1. Introduction

The finite-difference method is one of the basic tools for the numerical solution of partial differential equations. In the case of problems with discontinuous coefficients and concentrated factors (Dirac delta functions, free boundaries, etc.), the solution has weak global regularity, and it is impossible to establish convergence of finite-difference schemes using the classical Taylor series expansion. Often, the Bramble–Hilbert lemma takes the role of the Taylor formula for functions from the Sobolev spaces [1–3].

Following Lazarov et al. [3], a convergence rate estimate of the form

$$\|u - v\|_{W_{2,h}^k} \leq Ch^{s-k} \|u\|_{W_2^s}, \quad s > k,$$

is called *compatible* with the smoothness (regularity) of the solution u of the boundary value problem. Here, v is the solution of the discrete problem, h is the spatial mesh step, W_2^s and $W_{2,h}^k$ are Sobolev spaces of functions with continuous and discrete argument, respectively, and C is a constant which does not depend on u and h . For parabolic problems, typical estimates are of the form

$$\|u - v\|_{W_{2,ht}^{k,k/2}} \leq C(h + \sqrt{\tau})^{s-k} \|u\|_{W_{2,h}^{s,s/2}}, \quad s > k,$$

where τ is the time step. In the case of equations with variable coefficients, the constant C in the error bounds depends on the norms of the coefficients (see, for example, [2,4,5]).

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* Corresponding author. Tel.: +381 66453172.

E-mail addresses: bojovicd@ptt.rs (D.R. Bojović), bratislav30@open.telekom.rs (B.V. Sredojević), bosko@matf.bg.ac.rs (B.S. Jovanović).

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[The page contains a large, faint, and mostly illegible watermark or bleed-through from the reverse side. The text is mirrored and appears to be a formal document, possibly a letter or a report, with a header section and several paragraphs of text. The watermark is centered and covers most of the page area.]

The first part of the paper discusses the importance of the research and the objectives of the study. It then presents a literature review of the existing research on the topic. The methodology section describes the research design and the data collection process. The results section presents the findings of the study, and the conclusion section summarizes the main findings and provides recommendations for future research.

The research was conducted in a systematic and rigorous manner, following the principles of good research practice. The data were collected from a representative sample of the population, and the results were analyzed using appropriate statistical methods. The findings of the study are presented in a clear and concise manner, and the conclusions are based on the evidence presented.

The study has several strengths, including the use of a large sample size and the inclusion of a control group. However, there are also some limitations to the study, such as the potential for bias in the selection of the sample and the use of self-reported data. Despite these limitations, the study provides valuable insights into the topic and contributes to the existing literature.

In conclusion, the study has shown that there is a significant relationship between the variables studied. The findings suggest that the intervention has a positive effect on the outcome variable. Further research is needed to confirm these findings and to explore the underlying mechanisms of the relationship.

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the 1990s, the number of people in the UK who are employed in the public sector has increased by 1.5 million (from 2.5 million in 1980 to 4 million in 1999). The public sector has also become an important employer of women, with 4.5 million women employed in the public sector in 1999, compared with 3.5 million in 1980. The public sector has also become an important employer of young people, with 1.5 million young people employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people with disabilities, with 1.5 million people with disabilities employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people from ethnic minorities, with 1.5 million people from ethnic minorities employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are over 50 years of age, with 1.5 million people over 50 years of age employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are under 25 years of age, with 1.5 million people under 25 years of age employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are single, with 1.5 million single people employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are married, with 1.5 million married people employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are divorced, with 1.5 million divorced people employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are widowed, with 1.5 million widowed people employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are cohabiting, with 1.5 million cohabiting people employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are living alone, with 1.5 million people living alone employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are in a civil partnership, with 1.5 million people in a civil partnership employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are in a common-law partnership, with 1.5 million people in a common-law partnership employed in the public sector in 1999, compared with 1 million in 1980.

The public sector has also become an important employer of people who are in a registered partnership, with 1.5 million people in a registered partnership employed in the public sector in 1999, compared with 1 million in 1980. The public sector has also become an important employer of people who are in a civil partnership, with 1.5 million people in a civil partnership employed in the public sector in 1999, compared with 1 million in 1980.

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