

## STOCHASTIC MULTIVARIABLE SELF-TUNING TRACKER FOR NON-GAUSSIAN SYSTEMS

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This paper considers the properties of a minimum variance self-tuning tracker for MIMO systems described by ARMAX models. It is assumed that the stochastic noise has a non-Gaussian distribution. Such an assumption introduces into a recursive algorithm a nonlinear transformation of the prediction error. The system under consideration is minimum phase with different dimensions for input and output vectors. In the paper the concept of Kronecker's product is used, which allows us to represent unknown parameters in the form of vectors. For parameter estimation a stochastic approximation algorithm is employed. Using the concept of the stochastic Lyapunov function, global stability and optimality of the feedback system are established.

**Keywords:** ARMAX model, self-tuning tracker, non-Gaussian noise, robust statistics, global stability, optimality

### 1. Introduction

Adaptive control is a very important topic in control theory and practice. A vast amount of literature already exists on parameter estimation and adaptive control of stochastic systems (Åström and Wittenmark, 1989; Caines, 1988; Chen and Guo, 1991; Duflo, 1997; Goodwin and Sin, 1984; Kumar and Varaija, 1986). In those references it is assumed that stochastic disturbance has a Gaussian distribution. In some cases a dominant aspect in a control problem is the unmodeled dynamics, and then it is very important to assume the robustness of the control scheme (Ioannou and Sun, 1996; Landau *et al.*, 1998; Sastry and Bodson, 1989).

The problem of stochastic adaptive control of linear ARMAX systems has received considerable attention in the literature. In (Goodwin *et al.*, 1981), self-optimality and global stability for minimum variance regulation and tracking were proved. Self-optimality means that the time average value of the square of the tracking error is minimal. In (Becker *et al.*, 1985), for a stochastic gradient algorithm, the self-tuning property for the regulation problem was shown. This means that the adaptive control law converges to an optimal control law. The same results were obtained for the tracking problem in (Kumar and Praly, 1987). The results of (Lin *et al.*, 1985) show that the self-tuning regulation with the minimum variance cost criterion is asymptotically optimal. That does not occur for other cost criteria (for example, the quadratic cost criterion). An exception is the class of systems with large delays.

In the above papers it is shown that, in the case of the minimum variance problem, the closed-loop identifiability problem does not prevent self-tuning because every possible limit of parameter estimates leads to an optimal control law. Moreover, in (Becker *et al.*, 1985) it is shown that the parameter estimate converges to some random multiple of the true parameter. For consistent parameter estimation it is necessary to introduce an additional signal: continually disturbed control (Caines, 1988), a diminishing excitation signal (Chen and Guo, 1991), or an occasional excitation (Lai and Wei, 1986). The problem of the robustness of the minimum-variance controller is considered in (Praly *et al.*, 1989). It is shown that an adaptive controller for linear stochastic systems is optimal for all ideal plants and remains stable with respect to violations of the positive real condition and with respect to perturbations of the system in the graph topology from all ideal plants. In the case of multiplicative and additive system perturbations, the problem of adaptive control was considered in (Radenkovic and Michel, 1992). The underlying idea for the above problem is the construction of a suitable Lyapunov function for different periods of adaptation.

In this paper we will consider a minimum-variance controller when the disturbance is non-Gaussian. The non-Gaussian assumption introduces a nonlinear transformation of the tracking error in the estimation algorithm. A special case of such a situation is when one has *a-priori* information about the class of distributions to which the actual real disturbance belongs. In such a situation the theory of min-max estimation can be applied and the











