

# Recursive Estimation of the Takagi-Sugeno Models I: Fuzzy Clustering and the Premise Membership Functions Estimation

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*Fuzzy modelling is an approximation of nonlinear systems by a finite collection of linear systems. On this concept Takagi-Sugeno fuzzy models are based. The procedure for identification of these models include two steps: (a) estimation of membership functions, (b) model parameter estimation. In this paper only the step (a) is considered, where Gustafson-Kessel clustering algorithm is used. The algorithm detects clusters of different shapes. Parameter estimation of the premise membership function is based on the implementation of recursive least squares algorithm. Based on the obtained clusters, recursive least squares algorithm estimates parameters of membership functions. In this paper, it is assumed that the membership functions have triangular shape, performances of the proposed algorithm are demonstrated by simulation.*

**Keywords:** Fuzzy modelling, Fuzzy clustering, Nonlinear systems, Gustafson-Kessel algorithm

## 1. INTRODUCTION

In general, dynamical model of a system is nonlinear. Identification of this class of systems has been given many attention. Origins of this theory lie in different disciplines: control theory (identification of linear dynamical systems), nonparametric regression and statistics, learning theory, classification theory in pattern recognition, neural networks, fuzzy logic and other disciplines [1]. In this paper is considered the application of fuzzy logic for identification of nonlinear systems. Here will be discussed Takagi-Sugeno models [2]. In these models is used the idea of linearization of nonlinear systems in fuzzy regions of the state space. The structures are obtained, with several linear models. Input space is decomposed into a finite collection of fuzzy regions. The consequent functions describe system behavioural in those regions.

In classic control theory there are approaches that decompose nonlinear model into a finite collection of linear models. Example for that is included angle dividing method [3]. Using this method a finite collection of linear systems is obtained, as a base for further design of the controller. Similar, but more sophisticated methodology is obtained using gap metric concept [4], [5].

Methodologies [3]-[5], as well as the methodology discussed in this paper, are based on fuzzy logic, and they are alternatives to the well-known methodologies for design of controllers for nonlinear systems: feedback linearization [6] and backstepping [7].

The procedure for identification of Takagi-Sugeno models has two steps:

- Estimation of premise membership functions,
- Parameter estimation of consequent functions.

In this paper is discussed problem a), while problem b) will be discussed in complete authors' paper [8].

Problem a) is solved using cluster analysis on Cartesian product space of input and output. For cluster analysis is used Gustafson-Kessel fuzzy clustering algorithm. In order to complete the solution of the problem a), after the clusters are defined, it is necessary to

determine the parameters of membership functions. It is assumed that membership functions have triangular shape, and their parameters are estimated using recursive least squares algorithm.

The methodology exposed in this paper is demonstrated, thought simulation, on Hammerstein model.

## 2. TAKAGI-SUGENO MODELS

A nonlinear model  $y = f(x)$  can be expressed in the form of Takagi-Sugeno (TS) model based on input-output measurements  $\mathbf{u}_k = [u_{1k}, u_{2k}, \dots, u_{nk}]^T$  and  $y_k$  where  $k$  denotes measurements in the  $k$ -th moment, and  $n$  is the number of regressors in model.

TS model is a combination of logical and mathematical model. Logical rules are consisted of fuzzy premise, and consequent is a mathematical function. The general form of TS model [2]:

$$R_i : IF \mathbf{u} IS A_i(\mathbf{u}) THEN y_i = \mathbf{a}_i^T \mathbf{u} + b_i; \quad (1)$$

$$i = 1, 2, \dots, c$$

where,  $\mathbf{u} \in \mathbb{R}^n$  and  $y_i \in \mathbb{R}^1$  are inputs and outputs of the system, respectively. Values  $\mathbf{a}_i \in \mathbb{R}^n$  and  $b_i \in \mathbb{R}^1$  are parameters of TS model.  $R_i$  is the  $i$ -th rule, and  $c$  is the number of rules in rule base.  $A_i$  is multivariable premise membership function of the  $i$ -th rule.

For individual components of vector  $\mathbf{u}$ , TS model have the following form:

$$R_i : IF u_1 IS A_{i1}(u_1) AND \dots AND u_n IS A_{in}(u_n) THEN y_i = \mathbf{a}_i^T \mathbf{u} + b_i; \quad i = 1, 2, \dots, c \quad (2)$$

Degree of fulfilment of the rule is equal

$$\beta_i(\mathbf{u}) = \prod_{j=1}^n \mu_{A_{ij}}(\mathbf{u}) \quad (3)$$

where  $\mu_{A_{ij}}(\mathbf{u})$  is the membership function of the fuzzy set  $A_{ij}$ .







