

## DESIGN OF $H_\infty$ CONTROLLERS FOR MULTIVARIABLE DISCRETE GENERAL NONLINEAR SYSTEM

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**Abstract** – In this paper an explicit solution to the problem of disturbance attenuation with internal stability via full information feedback is considered. Design of  $H_\infty$  controllers for general nonlinear discrete-time model is presented. Solution is based on the concept of differential game, dissipation inequality and La Salle's invariance principle in discrete time. Given the sufficient conditions for solvability of the  $H_\infty$  control problem.

**Key words** – Discrete nonlinear systems,  $H_\infty$  controller, disturbance attenuation, internal stability

### I. INTRODUCTION

Design of  $H_\infty$  controllers is a central topic in control theory last ten years. The most number of results are devoted to design of  $H_\infty$  controllers for linear systems [1] – [4]. In many practical applications we must deal with nonlinear processes [5] – [8]. On the other hand computers, highly incorporated in the current practice, demand discrete models. Owing those facts it is a challenge for development of robust controllers for nonlinear process in discrete times.

In this paper we consider process which is described with discrete general multivariable model. The purpose of the paper is an explicit solution of the problem of disturbance attenuation with internal stability via full information feedback. Role of the controller is two – fold: to achieve closed loop stability and to attenuate with influence of the exogenous input on the penalty variable.

The problem of local disturbance attenuation with internal stability for continuous – time nonlinear system has been studied extensively in the number of recent papers [9] – [11]. Those papers discussed only the sufficient conditions for solvability of the  $H_\infty$  control problem. In the reference [10] studied necessary conditions for the nonlinear  $H_\infty$  – control problem to be solvable.

The approach to the discrete time nonlinear  $H_\infty$  control problem, in this paper, is based on the next three concepts:

- (i) dissipation inequality – based on theory of passive (or dissipative) systems [12]. Two key functions are storage function and supply rate function
- (ii) differential game – related to a particular type of discrete Hamilton – Jacobian equation [13] known as the discrete Isaacs equation [14].
- (iii) La Salle's Invariance principle in discrete time [15]

(iv) concept of  $L_2$  – gain [16]

Results of this paper is generalization of the results from reference [17] where the  $H_\infty$  theory for affine systems is considered.

Main results of this paper is formulated in the form of theorem. The proof of theorem is based on consideration of the Hamiltonian function associated with the  $H_\infty$  control problem, implicit function theorem, Schur complement and invariance principle. In this paper is considered full information feedback pattern.

### II. MATHEMATICAL TOOL

In this paper we will consider general discrete – time nonlinear system governed by the next equation

$$\begin{aligned}x_{n+1} &= f(x_n, u_n, w_n) \\ z_n &= z(x_n, u_n, w_n) \\ y_n &= y(x_n, w_n)\end{aligned}\quad (1)$$

where  $x \in R^n$  is the state,  $u \in R^m$  is the control input and  $w \in R^r$  is the disturbance signal which includes any external inputs to be tracked or any unknown disturbances to be rejected. The signal  $y \in R^p$  is the measured output, and  $z \in R^s$  is the penalty output which may include tracking error and a cost of the input  $u$  needed to achieve the prescribed control goals. We assume that

$$f: R^n \times R^m \times R^r \rightarrow R^n \quad (2)$$

$$z: R^n \times R^m \times R^r \rightarrow R^s$$

are smooth functions which satisfy  $f(0,0,0)=0$  and  $z(0,0,0)=0$ .

Now we will introduce necessary mathematical tool which will be utilized intuitively in solving the  $H_\infty$  control problem.

**A) Differential game.** We will consider a zero – sum, two – player, differential game described by the difference equation of the form

$$x_{n+1} = f(x_n, u_n, w_n) \quad (3)$$









