

## Recursive Estimation of the Takagi-Sugeno Models II: Estimation of Hammerstein Models

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**Abstract** – This paper considers the identification of a class of nonlinear systems. It is assumed that the model have block-oriented structure. The Hammerstein model will be considered within this model structure. Static nonlinearity is a polynomial function of the input signal. A linear part is described by discrete transfer function. The Hammerstein model is approximated with finite collection of linear models by Takagi-Sugeno model. In order to implement this model, it is needed to perform the fuzzy decomposition of the entire input signal space. Using Gustafson-Kessel algorithm and least squared method the membership functions are determined. Also, the least squared method is used for estimation of Takagi-Sugeno model.

**Key words:** nonlinear system, Takagi-Sugeno model, fuzzy clustering, Gustafson-Kessel algorithm, least squared method

### I. INTRODUCTION

In general, dynamical model of a system is nonlinear. Identification of this class of systems has been given much attention. The origins of this theory lie in different disciplines control theory (identification of dynamical systems), nonparametric regression and statistics, learning theory, classification theory in pattern recognition, neural networks, fuzzy logic and other disciplines [1]. In this paper, it is considered the application of fuzzy logic for identification of nonlinear systems. Here will be discussed Takagi-Sugeno models [2]. In these models it is used the idea of linearization of nonlinear systems in fuzzy regions of the state space. The structure of several linear models is obtained. Input space is decomposed into a finite collection of fuzzy regions. The consequent functions describe system behavioural in those regions.

In classical control theory there are approaches that decompose nonlinear model into a finite collection of linear models. Example for that is the included angle dividing method [3]. Using this method a finite collection of linear systems is obtained, as a base for further design of the controller. Similar, but more sophisticated methodology is obtained using the gap metric concept [4]-[5].

Methodologies [3]-[5], as well as the methodology discussed in this paper, are based on fuzzy logic, and they are alternatives to the well-known methodologies for the design of controllers for nonlinear systems: feedback linearization [6] and backstepping [7]

The procedure for identification of Takagi-Sugeno models has two steps:

1. Estimation of premise membership functions,
2. Parameter estimation of consequent function.

The first step is solved using cluster analysis on the Cartesian product space of input and output. For cluster analysis it is used Gustafson-Kessel fuzzy clustering algorithm, in order to complete the solution of the problem 1. After the clusters are defined, it is necessary to determine the parameters of membership functions. It is assumed that membership functions have a triangular shape, and their parameters are estimated using a recursive least squares algorithm. This step is discussed in [8].

After the premise membership functions are determined, the parameters of the consequent function can be estimated using least squared method. Consequent function can be any mathematical function, but for simplicity it is usually used an affine function.

In the identification of nonlinear systems, using fuzzy clustering, it is important to choose appropriate excitation signal. The choice of this signal largely depends on the problem, but the chosen signal must excite the entire range of amplitude and frequency. Fuzzy clustering algorithm can operate with noised signals, but with the increasing of the noise level, it is also increased the variance of estimated parameters [9].

The methodology exposed in this paper is demonstrated, thought simulation, on Hammerstein model. As the excitation signal it was used multi-sinusoidal signal, which is consisted of several sine waves of different frequencies and amplitudes. The simulation has shown good matching between the outputs of TS model and Hammerstein model.

### II. TAKAGI-SUGENO MODELS

A nonlinear model  $y = f(x)$  can be expressed in the form of Takagi-Sugeno (TS) model based on input-output measurements  $\mathbf{u}_k = [u_{1k}, u_{2k}, \dots, u_{nk}]^T$  and  $y_k$  where  $k$





