

Stochastic Adaptive Control Using the Robust Least Squares Algorithm

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This paper considers properties of the Astrom-Wittenmark self tuning tracker for MIMO systems described with the ARX model. It is supposed that the stochastic noise has the non-Gaussian distribution (condition always present in practice). The consequence of that fact is a nonlinear transformation of the tracking error in the direct adaptive minimum variance controller. The system under consideration is the minimum phase with different dimensions for input and output vectors. Using the concept of the Kronecker product it is possible to represent unknown parameters in the form of a vector. The tensor calculus is thus avoided. Global stability is proved without any modification of the matrix gain in the recursive algorithm. The paper also discusses the relation of the assumption about the absolutely continuous finite-dimensional distributions and different modifications of a high-frequency gain. The paper presents theoretical results but the adaptive control methodology has already been present for many years in military systems (CH-47 helicopter and X-15 aircraft).

Key words: adaptive control, ARX model, non-Gaussian disturbance, self-tuning tracker, system stability, least squares method.

Introduction

THE analysis of adaptive controllers is a very important topic in the control area [1]. In this reference it is shown that if the least squares parameters estimates converge to some limit then the adaptive controller must be optimal but, as noted, it is very difficult to prove that the estimates are indeed convergent. After that much attention has been drawn to establishing the global stability and the asymptotic optimality for adaptive controllers. Significant progress in this direction was made in [14] where global convergence has been established for a class of stochastic adaptive control algorithms based on the stochastic approximation method. The next important step is a result presented in [23]. Namely, from the practical point of view, least squares generally have a superior rate of convergence in comparison with the stochastic approximation algorithm. But, in that case, it was necessary to modify the gain matrix for the global convergence of algorithms. In [17] an attempt was made to remove the above restriction. For a minimum phase system where adaptive noise is i.i.d. and Gaussian, using the Bayesian embedding method and the properties of normal equations, a least squares-based adaptive tracker converges outside an exceptional set of the Lebesgue measure zero in the parameter space. In this approach the restrictions are: Gaussianity and independency of noise and the exceptional set. Very important results are presented in [13] where the Astrom-Wittenmark self-tuning regulator and the ELS-based adaptive tracker are considered. It is shown by a careful analysis of growth rates how to avoid the need to establish parameter convergence. Also, convergence of the original Astrom-Wittenmark self-tuning regulator is proven rigorously. Using the ideas from [17] and [13], reference [21] presents a more comprehensive theory of

stochastic adaptive filtering, control and identification. It is also established that the parameters converge to the null space of a certain matrix. The results from [13] are used for some problems in the model reference adaptive control [20]. Weighted estimation and tracking for a multivariable ARMAX model is considered in [2]. This paper introduces a random weighting sequence and shows that the given algorithm has the performance of the ELS for the strong consistency and matches the best result of SG for the adaptive tracking. Some aspects of tuning of self-tuning controllers is discussed in [24]. A further important step is reference [11] where the best convergence rate of self-tuning regulators (logarithmic law of STR) is found. The overview of adaptive methodology is given in [12] and [16]. This paper will consider the Astrom-Wittenmark controller when the disturbance is non-Gaussian. The non-Gaussianity introduces nonlinear transformation of the tracking error in the estimation algorithm. A special case of such situation is the case when there is an priori information about the class of distribution to which the real disturbance belongs. In such situation the theory of min-max estimation can be applied and so the given algorithm is known as a robust algorithm.

Reference [6] considers the robust SG algorithm (nonrobust version of SG algorithm is considered in [4] and [18]) as well as the stability and optimality of the minimum variance controller. The parallel result for the ELS algorithm for SISO systems described by the ARMAX model is presented in [8]. It is shown that for the stability of the adaptive controller no modification of the gain matrix is necessary. A tracking problem when the noise is non-Gaussian and when unmodeled dynamics is also present is considered in [7]. Robust predictor for SISO systems is presented in [9].

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