

# Robust Recursive Identification of Multivariable Processes

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*Many industrial processes have the multivariable nature (boiler plant, evaporators, distillation columns, etc.). One of the key requests of production is energy saving and product quality improvement (these two categories are tightly connected). In order to achieve this it is necessary to carefully design process control strategies. Therefore, the quality mathematical model of the process is needed. In this paper it is assumed that the process is described with multivariable ARX (AutoRegressive model with eXogenous input) model. The key assumption, justified with numerous studies of real processes, is that the stochastic disturbance has non-Gaussian distribution. That fact affects on form of recursive identification algorithm. The algorithm becomes nonlinear. Simulations are performed that demonstrate the superiority of the proposed algorithm over the standard algorithms.*

**Keywords:** Multivariable systems, ARX model, Non-Gaussian distribution, Robust recursive identification

## 1. INTRODUCTION

The task of identification is estimation of unknown dynamics based on measurement data. This is a key ingredient for areas of adaptive control and adaptive signal processing. Theory of identification covers wide range of problems [1]-[5]. New impulse in the development of the theory is given by the area of statistical learning theory [6]-[9].

Multivariable systems represents very important class of systems in practice. Special attention is devoted to their identification [10]-[13]. In this area, the problem of identification is considered in the deterministic framework or with the assumption that the stochastic disturbance has a Gaussian distribution. Intense practical studies [14]-[15] have not shown justification of the assumption of normal distribution of disturbances. Namely, in population of observations there are rare large observations and the result is that stochastic disturbance has a non-Gaussian distribution. As a result, the efficiency of the identification algorithm based on the assumption of the Gaussian distribution of disturbances is reduced. Because of this, a great effort has been invested for the synthesis of robust identification algorithms that have low sensitivity to changes in the disturbance distribution. The fundamental contribution, in this sense, was given by Huber [16-17] who laid the foundations of an area known as robust statistics. This theory, rather than the assumption of exactly known distribution of the stochastic disturbance, introduces the assumption of a priori known distribution class to which the disturbance belongs. The application of these ideas in system identification is exposed in [18-20], and in adaptive control in [21-22].

A single-input single-output (SISO) system is considered in references [18-22]. In this paper, the robust recursive identification of multivariable ARX models is considered. It is assumed that the classes of probability distributions, to which stochastic disturbances and unknown parameters of the dynamic system belong, are known. A priori information on the disturbances introduces a nonlinear transformation of the prediction error, which makes the nonlinear recursive algorithm, and a priori information on the parameters defines the initial

conditions of the algorithm for the parameter vector and the gain matrix.

At the end of the paper, simulations that illustrate the behaviour of the robust identification algorithm are presented.

## 2. MULTIVARIABLE ARX MODEL

Assume that the considered system is described with linear multivariable ARX model with  $r$ -dimensional input and  $p$ -dimensional output

$$A(q^{-1})y_k = B(q^{-1})u_k + w_k \quad (1)$$

where  $A(q^{-1})$  and  $B(q^{-1})$  are matrix polynomials in the shift-back operator  $q^{-1}y_k = y_{k-1}$ . Degrees of polynomials  $A(q^{-1})$  and  $B(q^{-1})$  are  $n$  and  $m$ , respectively

$$A(q^{-1}) = I + A_1q^{-1} + \dots + A_nq^{-n} \quad (2)$$

$$B(q^{-1}) = B_1q^{-1} + \dots + B_mq^{-m} \quad (3)$$

where  $A_i (i = 1, 2, \dots, n)$  are  $p \times p$  matrices, and  $B_i (i = 1, 2, \dots, m)$  are  $p \times r$  matrices.

The stochastic disturbance  $\{w_k\}$  is a martingale-difference in relation to the nondecreasing family of  $\sigma$ -algebras  $\{F_k\}$ .

Unknown matrices' coefficients are

$$(\theta^M)^T = [A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m] \quad (4)$$

Now model can be written in following form

$$y_k = (\theta^M)^T X_k + w_k \quad (5)$$

where

$$X_k^T = [-y_{k-1}^T \dots y_{k-n}^T u_{k-1}^T \dots u_{k-m}^T], \quad X_k \in \square^{(np+mr) \times 1} \quad (6)$$

Let us introduce the matrix

$$\Phi_k = \begin{bmatrix} X_k^T & 0 \\ & \ddots \\ 0 & X_k^T \end{bmatrix} = I \otimes X_k^T \quad (7)$$

where  $\otimes$  denotes Kronecker product.









