

The Generator of Markov Chain Estimation in Hybrid Stock Models Using Pseudogradient Algorithm

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Abstract: *Very new research in field of financial engineering introduce the control theory. For example, the portfolio selection is a typical problem of stochastic optimal control. In this paper we study a class of hybrid models for the stock markets where presents coexistence of continuous dynamics and discrete events. In the original geometric Brownian motion models for price of the stock, we introduce, for both the rate of return and the volatility in the hybrid model, dependence on a continuous-time Markov chain. This model can deal with random volatility by incorporating market trend with other economic factors. The approach in the paper requires estimation algorithm for the generator of the underlying Markov chain. For that we generalize the Kiefer-Wolfowitz procedure by using the concept of pseudogradient. By using the results of estimation, one can then proceed to make equity liquidation decisions.*

Key words: *Geometric Brownian motion, hybrid models, Markov chains, pseudogradient, estimation*

I. INTRODUCTION

In this paper we will model stock market as hybrid systems. In such systems the continuous dynamics and discrete event coexist. The Black-Scholes model, based on geometric Brownian motion (GBM), has been widely used in analysis of options pricing and portfolio management [1]-[3]. The GBM model uses a stochastic equation with deterministic expected return and nonrandom volatility. Because, they are not responsive to the random fluctuation on financial market and, also, not suitable for a longer horizon. Owing that facts, it is desirable to modify the model so as to capture the random parameter changes such random volatility [4].

The stock market is time varying and has discontinuities caused by uncertain and exogenous discrete events. The price of individual stock is effected by market trends, interests rates, business cycles, etc. In this paper we will use hybrid models for description of above situations. In this way we can get switching Black-Scholes model. A switching GBM model (a collection of GBM models modulated by a hidden Markov chain with finite state space) is considered in reference [5]. Related norm on hedging of options is presented in [6]. For hybrid market models, when the generator of the hidden Markov chain is specified, the system behavior is determined [7], [8]. Then it is possible to make decision on portfolio selection or stock liquidation. The stochastic optimization procedure for such task is considered in [9] where convergence is proven and established the rate of convergence.

In recent years to control theory becomes important tool in the quantitative finance problems. In reference [10] the problem of dynamically hedging a basket option on multiple underlying stocks and in the presence of proportional transactions costs is formulated as a linear quadratic control problem with the constrains. The reference [11] deals with the problem of reconstructing the tree-like topological structure of a network of linear dynamical systems. It is, also, considered the application of the techniques to the analysis of an actual complex network, i.e., to high frequency time series of the stock market. The new paradigm for the trading of equities is proposed in [12]. In that reference the control corresponds to a feedback law which modulates the amount invested in stock over time. The controller includes a saturation limit corresponding to a limit on the value at risk. Further development is presented in [13] where developed an algorithm for triggering a trade. Consideration a multi-sector model of economy is presented in [14]. In the paper [15] the robust stochastic approximation is proposed for improvement results of Black-Scholes model in the long run time period. It is first developed a robust recursive stochastic approximation algorithm to estimate implied volatility. After that, the estimated parameters to price options is used.

In this paper we study a class of hybrid models for the stock market. In the original geometric Brownian motion models for price of the stock, we introduce for both the rate of return and the volatility in the hybrid model dependence on a continuous-time Markov chain. The main task is the estimation of the underlying Markov chain. For that we generalize the Kiefer-Wolfowitz procedure [16] by using the concept of pseudogradient [17], [18]. The procedure is improved using a priori information about the options prices. The a priori information is given in the form of fiducial probability [19]. Also, we introduce the dependence of approximation gain from the observations. The proposed recursive procedure is the most general procedure of this type. The procedure includes apriori information about the stochastic noise and, also, about the unknown parameters. Such procedures are robust and have high speed of convergence. Owing that features procedures are suitable for applications especially in situation when present stochastic variations of financial market. In this paper developed algorithm are combined with hybrid stochastic models.

