

## Outlier Robust One-Step-ahead Adaptive Predictor for Hammerstein Models

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**Abstract** – In paper considered outliers-robust recursive stochastic approximation algorithm for adaptive prediction of MIMO (multiple-input multiple output) Hammerstein models. The static nonlinear block has polynomial form and linear block is output-error model. It is supposed that a priori known class of distributions to which belongs the real disturbance. In that situation we can use Huber's methodology for design of robust algorithm which introduces nonlinear transformation of prediction error. Model transformation allows representation of unknown matrix parameters in the form of vector. The problem is not considered before in the field of adaptive prediction. Simulation study presents the practical behaviour of algorithm.

**Key words:** Hammerstein model, outliers, prediction, stochastic approximation

### I. INTRODUCTION

Output error (OE) models are frequently used for identification, prediction and adaptive control system. This paper focuses on the synthesis of one-step-ahead predictors, for MIMO non-linear systems. The non-linear model discussed in the paper belongs to class of block-oriented models. The example is a Hammerstein's model where static non-linear block is given in polynomial form while linear model is OE model.

Adaptive prediction focuses on recursive estimation of future values of system outputs based on past and present values of system inputs and outputs [1]. Applications of prediction algorithms are numerous [2]-[3]. Adaptive prediction in this work is based on algorithm of stochastic approximation. Standard algorithms of adaptive prediction assume that distribution of stochastic disturbance is exactly known (usually it is Gaussian distribution). However, analysis of practical measurements has shown that, in population of observations, exists rare large observations (outliers) [4]. Direct implication is that assumption on exact knowledge of disturbance distribution should be replaced by the assumption on a priori knowledge of the class of distribution to which the relevant disturbance belongs. The theory of robust statistics is built on this basis [5]-[6]. Based on this theory it is possible to get, in the statistical sense, robust recursive algorithms (reduced sensitivity to change of the disturbance distribution) for estimation of the parameters of dynamic phenomena. The application of the above mentioned ideas in the problems of identification and predictions was demonstrated in references [7]-[11] and [17]-[22]. Simulations have shown superiority of robust algorithms in relation to classical (linear) algorithms.

In regards to algorithms based on stochastic approximation it is necessary to invest additional effort for increase of speed of algorithm convergence (especially on initial iterations).

As far as authors are informed, new robust recursive algorithm for prediction of non-linear MIMO OE models is proposed in this paper. Namely, algorithm is modified so that the unknown parameters are given in the form of vector. For approximately normal distributions non-linear transformation of prediction error is Huber's function and algorithm gain is exactly defined for this case (matrix M in algorithm) which, as well, represents new result. This was achieved by the application of Laplace functions. [12].

The algorithm developed in this paper is generalized form of algorithm for identification of MIMO systems when unknown parameter has vector form [14].

### II. ROBUST RECURSIVE STOCHASTIC GRADIENT ALGORITHM

Suppose that the considered system is described by the nonlinear multivariable output-error (OE) model with r-dimensional input and p-dimensional output:

$$y_k = q^{-1} F^{-1}(q^{-1}) B(q^{-1}) f(u_k) + w_k \quad (1)$$

where  $B(q^{-1})$  and  $F^{-1}(q^{-1})$  are matrix polynomials and  $q^{-1}$  denotes the shift-back operator ( $q^{-1}x_k = x_{k-1}$ ). Orders of polynomials  $B(q^{-1})$  and  $F^{-1}(q^{-1})$  are  $m$  and  $n$ , respectively

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \dots + B_m q^{-m} \quad (2)$$

$$F(q^{-1}) = I + F_1 q^{-1} + \dots + F_n q^{-n} \quad (3)$$

where  $B_i (i=1,2,\dots,m)$  are  $p \times r$  matrices, and  $F_i (i=1,2,\dots,n)$  are  $p \times p$  matrices. The stochastic disturbance  $\{w_k\}$  is a martingale-difference in relation to the nondecreasing family of  $\sigma$ -algebras  $\{F_k\}$ . The Hammerstein model is given on the Figure 1.





