

A UNIFIED APPROACH FOR DESIGN OF H_∞ CONTROLLERS

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Abstract - This paper considers design of H_∞ controllers for minimum phase and unstable processes. It is supposed that plant is described with single-input single-output DARMA model. For controller design is used Nevanlinna-Pick interpolation theory. Structure of controller is very simple and tool for design is theory of polynomial equations. Design is unified in the sense that valid for both: continuous and discrete control systems thanks to theory of delta operators. Form of stabilizing controllers is dual to the case of nonminimum phase and stable processes.

Key words - Interpolation, delta operators, polynomial equations, H infinity

I. INTRODUCTION

Dominant approach in the control theory during the seventies was the LQG design [1]. The main problems in the LQG are disturbance attenuation and measurement noise abatement. Explicitly incorporation plant uncertainty into control problem is not performed. Sometimes is used differential sensitivity of the closed loop systems to variations in the plant [2]. Differential sensitivity analysis often gives a good prediction of the changes that occur in the closed loop system when the plant changes by a moderate amount. Design for such cases is robust. But differential sensitivity specifications cannot guarantee that the closed loop system does not become unstable when the plant changes by a non-vanishing amount.

In the early eighties researchers reestablished the link to the classical work of Bode and Nyquist by formulation a tractable mathematical notion of uncertainty in the input-output and developed mathematical tool to cope with it [3]. Early reference on modeling and assessing the effects of uncertainty relies on the small gain theorem and bounds on system norms [4].

The robustness specifications, in contrast to the differential sensitivity specifications, introduce the sizes of plant variations explicitly and limits the worst case change in the closed loop system that can be caused by one of the possible plant perturbations [5].

Essential part of the LQG design is Kalman filter which requires excessive prior knowledge of the nature of the disturbance. This assumption is equivalent to knowing the noise spectral density function. An alternative starting point would be to consider all possible noise sources of restricted

energy. That is base for introduction H_∞ norm for signal and systems and the key point for H_∞ controller design [5], [6], and [7].

Above references deal mainly with the H_∞ controller design in the continuous domain. The most industrial controllers are digital and that is strong motivation for adapting design techniques from continuous time to discrete time. In [5] is considered two approaches for design of H_∞ controllers. The first is based on discretization of continuous plant and then design the controller in discrete time or in the design controller in continuous time and then discrete it. The second approach is a direct attack in the continuous-time domain, where sampled-data system are time-varying (actually, periodic). Reference [8] deals with the design of discrete H_∞ controller in state space domain. Polynomial approach for H_∞ controller design, using game theory, is presented in [9].

In this paper we consider process which is described with DARMA (deterministic ARMA) model [10]. Process is minimum phase and unstable. Design of H_∞ controller is based on delta operators and interpolation theory. Delta operators offers the some flexibility as the shift operators in the description of discrete systems and, also, has some advantages: (i) it allows a unified system theory to be developed for continuous and discrete systems, (ii) most continuous time results can be given as a special case of the discrete results (by setting the sampling period to zero), (iii) it has numerical advantages in most cases of practical interest. Comprehensive treatment of continuous and discrete control and estimation theory in the unified fashion is presented in [11].

Key mathematical tool for design of H_∞ controller, in this paper, is interpolation theory [12] which is based on analytic function [13]. The theory offers extremely simple procedure for controller design in the comparison with the published results [5], [6], [7] and [9]. In this paper is used Nevanlinna-Pick theory. First is found the set of all stabilizing controllers in the form of polynomial equations. Parametrization of controllers is dual for the nonminimum and stable process [15]. Feedback system is internally stable and output of the system track a step input with the ϵ -accuracy. Structure of the given H_∞ controller is extremely simple.

On the end of introduction shortly will be discussed related results. In [16] problem of discrete H_∞ controller for stable

