

## About Some Open Problems in Control Theory

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**Abstract** – In this paper we will consider a few open problems in control theory. Those problems, also, very important from the practical point of view. We will review next problems: (i) the simultaneous stabilization problem of finding a single controller which stabilizes a finite set of different plants, (ii) stabilization with controllers which have fixed structure, (iii) the possibilities of nonstationary feedback. We will consider in detail the first problem. It is well known that solution for such case exists only for collection of two systems. Using recently published results about the extended superstability we will solve problem for general case for continuous-time control systems. The problem is reduced to solve the linear inequalities.

**Key words:** open problems, simultaneous stabilization, extended superstability

### I. INTRODUCTION

The control theory of linear systems has a high level of maturity. But, in the frame of that theory else have important number of unsolved problems. Much of them are presented in the books [1]-[3]. In the sequel we will consider three problems cited in the abstract.

The simultaneous stabilization problem consists of finding a single controller which stabilizes a finite set of different plants. One commonly cited application is the desire to control a system under normal operating conditions as well as under several different failure modes, each represented by different system description [4]. Similarly a linear controller could potentially stabilize the plants linearized about several different points of operation. Simultaneous stabilization is subtopic of robust control. Robust stabilization simultaneously stabilizes a continuous range of plants, whose parameters lie within predefined regions, subject to possible performance constraints. The major distinction between robust and simultaneous stabilization is in the number of plants which each attempts to stabilize. Robust stabilization contends with infinite (uncountable) number of plants, whereas simultaneous stabilization only deals with a finite number. Nevertheless, the simultaneous stabilization of finite number of the system is deceptively different. Unlike robust stabilization in which the continuum of plants must not vary too far from a nominal plant, there may be no assumptions on the interrelatedness of the finite number of distinct plants. To date, there is complete tractable solution to the simultaneous stabilization problem only when there are no more than two plants [5], [6].

The second problem is the system stabilization with fixed structure controllers. In the modern optimal control ( $H^\infty$  and  $L^1$ ) the order of the controllers are very high and that fact can create the difficulties in applications. Because it is extremely important to have controllers with low

orders. The very important practical example is PID controller. It is well known that by PID controller we can stabilize the plant of order two. Extremely important question is: Can we stabilize the plants with orders greater than two with PID control algorithm? Now it is known that stable process can be stabilized with P controller whereby gain of controller is small [7]. Also, the minimum-phase processes can be stabilized with high gain controller [8]. Using D-decomposition [9] it is possible to stabilize system with arbitrary order by PI controller. And that is all. For controllers with three or more parameters the answer is unknown and very difficult.

The third problem is devoted to nonstationary feedback and formulated by Brockett [1]. In [10] it is shown that nonstationary feedback superior than stationary feedback (fixed gain). Two cases are considered

- (i) low frequency stabilization based on Poincaré-mapping
- (ii) high frequency stabilization based on the averaging theory

For both cases necessary and sufficient conditions are proved.

### II. STABILIZATION WITH FIXED STRUCTURE CONTROLLERS

Let us suppose the process is described with

$$G(s) = \frac{A(s)}{B(s)}, \deg A(s) = m, \deg B(s) = n, m \leq n \quad (1)$$

We put next question: Is it possible to stabilize process (1) with controller

$$C(s) = \frac{N(s)}{M(s)} \quad (2)$$

if degrees of polynomials  $N(s)$  and  $M(s)$  is smaller from fixed numbers. From (1) and (2) one can get characteristic polynomial for closed-loop system.

$$P(s) = A(s)N(s) + B(s)M(s) \quad (3)$$

Question now is: Can we choose polynomials  $N(s)$  and  $M(s)$  is Hurwitz polynomial.

We have solution for next two cases





