

Robust Stabilization Using Quantized State Feedback

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This paper studies the problem of robust stabilization for uncertain systems via quantized feedback. Control with limited rates includes ideas from both control and information theory. The rate constraint introduces quantization into feedback. In this paper we assume that communication channel connects sensor and controller. We suppose that the communication network has up-link structure i.e. encoder-channel-decoder-controller. We will consider linear time-discrete-time systems with unmodeled dynamics. The unmodeled dynamics is described with affine family of matrices. When noise is present we will determine the minimum bit-rate above which stabilization can be achieved. It is shown that presences of unmodeled dynamics increase the lower bound of bit-rate which guarantee stability of the system. In the second part of the paper we consider variable-step quantization. For such case, in the presence of disturbance, we prove that state of system remains uniformly bounded.

Keywords: discrete system, communication network, quantization, bit-rate, stability

1. INTRODUCTION

Long time in engineering systems with large communication bandwidth the communication and control considered as independent functions. In such situation the analysis and design of overall system is simplified. But recent applications in sensor networks [1], microelectromechanical systems [2], mobile telephony [3] and industrial systems [4] introduced the new challenge for investigation. The essence is to control one or more dynamical systems using multiple sensor and actuators transmitting and receiving information over a digital communication network. Because the understanding control over communication network is extremely important topic for future research [5].

The total communication capacity of communication network may be large but each component can available only a small portion. That can introduce large quantization errors which can be modeled as extra additive white noise. Such approach is valid if the quantization resolution is high. If the open-loop dynamic is unstable and resolution is coarse such approach is not correct [6]. Recently is showed [7] that result in [8] on quadratic stabilization of linear systems using quantized state feedback with coarsest quantization density can be obtained from the quantization error as sector-bounded uncertainty.

In the constrained communication problems the two question are very important. The first one is: what is the smallest feedback data rate above which a given dynamical system can somehow be stabilized? This is analogous to Shannon's source coding theory which determines the smallest data rate above which a given random process can be reliably communicated [9]. The main difference is that in control systems the data is not only transmitted from one

point to another but are used in a feedback loop. The second important question is: for a given dynamical system which fundamental trade off must exists between the communication rate and optimal attainable control performance. This is the control theoretic version of Shannon's rate distortion theory [9].

In [6] was shown that a noiseless and unstable linear plant with eigenvalues less than 2 in magnitude can be asymptotically controlled to the origin using memoryless quantization of the state. If an eigenvalues magnitude was larger than 2 then chaotic trajectories appeared. The first result on minimum data rate for stabilizability appeared in [10]. In [11] is determined data rate for which the state is bounded for memoryless quantizer. The improvement from boundedness to asymptotic stability is based on quantizer which possess memory and follow adaptive zooming-in / zooming-out strategy [12].

In the research literature we can differentiate three type of quantizers:

- (i) Uniform quantizer [11]
- (ii) Logarithmic quantizer [8]
- (iii) Chaotic quantizer [13]

In the framework of memoryless quantization a recent result [14] is very promising.

In the literature a number of results on noisy channels have been proposed: (i) digital erasure channel [15], (ii) binary symmetric channel [16] and truncation concept channel [17]. In last reference the novel concept of anytime capacity is introduced. Such concept takes into consideration the recursive structure of incoming data.

Result of this paper close to result presented in [11] and [18]. The main difference is in consideration of linear discrete-time systems and variable step quantizer. In the first part of the paper we will find minimum bit-rate for which stabilization of discrete-time noiseless process, in the presence of uncertainty, is possible. After that we will consider the design of encoder-decoder pairs when the measurements are noisy. In the paper the linear predictor is designed so that the prediction error rather than the complete signal is quantized and send to the receiver. Here the variable step quantization is used. From the results of this paper follows that presence of uncertainty increases bit-rate which is need for system stability. The uncertainty is modeled as a affine family of matrices. Also, possible to use other models for uncertainty.

