

## EXPONENTIAL STABILITY OF NONLINEAR HYBRID SYSTEMS

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**Abstract:** *This paper proposes a method for analysis of exponential stability of analysis of deterministic nonlinear switched systems. The system models belong to a finite set of models. It is assumed that there is no jump in the state at switching instants and there is no Zeno behaviour, i.e. there is finite number of switches on every bounded interval. The hybrid system have wide applications; modeling of communication network, system with quantization, networked control systems and process control. For stability analysis the multiple Lyapunov function is used. It is considered autonomous nonlinear switched systems. The exponential stability is practically important Because such systems are robust to perturbation. The globally exponentially stability is proved.*

**Key words:** *Switching systems, multiple Lyapunov function, exponential stability.*

### 1. INTRODUCTION

Hybrid systems are digital real-time systems which are embedded in analog environments. Analog part of the hybrid systems is described with differential equations and discrete part of the hybrid systems is a event driven dynamics which can be described using concept from discrete event systems [1] – [2].

In this paper we will consider the switched systems which can be viewed as higher-level abstractions of hybrid systems [3]. The subsystems of the switched system are modeled as nonlinear deterministic differential equations.

In this paper we consider the exponential stability of switched deterministic systems. There are two ways to analyze stability of switched deterministic systems. The first one is construction of common Lyapunov function. Find the common Lyapunov function is a difficult task [4]. The second one utilizes multiple Lyapunov function [5]. Here we will use the second approach. We assume that

A) there is no jump in the state  $x$  at the switching instants

B) there is no Zeno behavior, i.e. there is a finite number of switches on every bounded interval of time.

The situation with jump in the state of  $x$  at the switching instants is considered in [6] and [7]. The hybrid control is an important tool for large class of real problems. That type of control systems cover the system with quantization [8], control of wireless network [10] and networked control systems [9]. Switching control strategy in [11] is used for control of time-delay system. Namely, here the concept of multiple models and concept of switching controllers is used.

The analog part of system is describe by finite set of continuous-time models with input delays and

unmodeled dynamics in the form of affine family. As a result, the LQ switching controllers are given. In the [12] specific form of hybrid LQ controllers is derived using linear matrix inequalities.

The mathematical model for real process generally has the Hammerstein-Wiener form. It means that on the input and output of the process present nonlinear elements (actuator and sensor). In reference [13] the Hammerstein model (has only input nonlinearity) is considered. The controller for such case is combination of piecewise lineae control (PLC) with low-and-high gain feedback law (LHG). The PLC control has associated switching surfaces in form of positively invariant set. The key features of PLC/LHG controllers is that the saturation level of control signal avoided.

In reference [14] the stochastic nonlinear hybrid sistem is considered. The exponential m-stability is proved. The main results of the paper are:

(i) the exponential  $m_1$ -stability os stochastic switched system whereby  $m_1 \in (0, m)$  ;

(ii) the stability in probability

The presence of analog uncertainty is considered in [15] and [16].

The syrvey of recent results from theory of hybrid system is presented in [17]. A necessary and sufficient condition for asymptotic stabilizability of switched linear system is described.

In this paper we find a set of conditions under which the deterministic switching systems is exponentially stable. We use the multiple Lyapunov functions approach. The finite set of models is nonlinear. It is important to mention that exponentially stable equilibria is relevant for practice. Namely, such systems are robust to perturbation.





