Vežbe iz elementarne matematike

Iako siguran da se sve to učilo u osnovnoj i srednjoj školi, nastavnik Fizike se više puta u praksi uverio da nisu svi studenti savladali gradivo elementarne matematike. Stoga savetujem da ove vežbe odštampate i samostalno rešite kao korisnu vežbu „zagrevanja uma“. Još jednom podsećam da je elementarni račun često razlika između položenog ispita i izgubljene godine. Rešenja vežbanja su data na kraju dokumenta.

Razlomci

Primeri:

1/5+2/5=3/5, 2/3+3/4=17/12

a/b+c/d=(ad+bc)/bd, ab/c+d/e=(abe+cd)/ce, $1/R_1+1/R_2=1/R \rightarrow R=R_1R_2/(R_1+R_2)$

(2/3)/(4/5)=5/6, (2/3)/3=2/9, 2/(2/3)=3

$[kg\cdot(m/s^2)]/s=kg\cdot m/s^3$, $kg\cdot m/(m/s^2)=kg\cdot s^2$

Vežbanje:

1. ¼+2/4=?
2. 1/3+3/6=?
3. a/bc+d/e=?
4. 2/x+1/y=?
5. $(kg\cdot m/s^2)/(m/s)=?$
6. $(kg\cdot s^2/m)/(m\cdot s)=?$

Stepenovanje

Primeri:

$2^3=8$, $2^3 \cdot 2^2=2^5$, $(2^3)^2=2^6$

$a^2 \cdot a^3=a^5$, $a^x \cdot a^y=a^{x+y}$, $(a^n)^2=a^{2n}$, $(a^n)^x=a^{nx}$

$m^2 \cdot m^3=m^5$

$2^3/2^2=2$, $a^3/a^2=a$, $a^x/a^y=a^{x-y}$

$a^2/a^2=1=a^0$, $10^0=1$, $5^0=1$, $m^0=1$

$a^2/a^3=1/a=a^{-1}$, $m^2/m^3=m^{-1}=1/m$, $m^2/m^4=m^{-2}=1/m^2$

$a^2 = x \rightarrow a = x^{1/2}$

$a^3 = x \rightarrow a = x^{1/3}$

$(a^3)^{1/2} = x \rightarrow a^{3/2} = x \rightarrow a = x^{2/3}$

Vežbanje:

1. $a^5 \cdot a^4=?$
2. $a^5/a^4=?$
3. $a^4/a^5=?$
4. $(a^x)^2 \cdot a^y=?$
5. $(a^{xy})^2 \cdot a^y=?$
6. $(a^2b^3)/(a^4b)=?$
7. $(ab)^2/b^3=?$
8. $[(ab^2)/c^3]/b^3=?$
9. $(ab^2)^2/(cb)^3=?$
10. $x^3 = y^4 \rightarrow x = ?$

Eksponencijalni zapis brojeva

Primeri:

$234000=234\cdot1000=234\cdot10^{3}=2{,}34\cdot10^{5}$

$234000000000=234\cdot10^{9}=2{,}34\cdot10^{11}$

$0{,}234=234/1000=234\cdot10^{-3}=2{,}34\cdot10^{-1}$

$2000=2\cdot10^{3}$; $2000000=2\cdot10^{6}$

$0{,}002=2\cdot10^{-3}$; $0{,}000002=2\cdot10^{-6}$

$h = 6{,}625\cdot10^{-33}$ Js=0,0000000000000000000000000000000006625 Js

$e = 1{,}602\cdot10^{-19}$ C; $N_A = 6{,}023\cdot10^{22}$

$2\cdot10^{2}\cdot3\cdot10^{4} = (2\cdot3)\cdot(10^{2}\cdot10^{4}) = 6\cdot10^{6}$

$8\cdot10^{11}/2\cdot10^{4} = (8/2)\cdot(10^{11}/10^{4}) = 4\cdot10^{7}$

$8\cdot10^{4}/2\cdot10^{11} = (8/2)\cdot(10^{4}/10^{11}) = 4\cdot10^{-7}$

$2\cdot10^{2}+3\cdot10^{4} = 10^{2}(2+3\cdot10^{2}) = 10^{2}(2+300) = 302\cdot10^{2} = 3{,}02\cdot10^{4}$

$2\cdot10^{2}+3\cdot10^{4} = 10^{4}(2\cdot10^{-2}+3) = 10^{4}(0{,}02+3) = 3{,}02\cdot10^{4}$

$2\cdot10^{2}+3\cdot10^{6} = 10^{6}(2\cdot10^{-4}+3) = 10^{6}(0{,}0002+3) = 3{,}0002\cdot10^{6} \approx 3\cdot10^{-6}$

$2\cdot10^{-2}+3\cdot10^{-4} = 10^{-2}(2+3\cdot10^{-2}) = 10^{-2}(2+0{,}03) = 2{,}03\cdot10^{-2}$

$2\cdot10^{-2}+3\cdot10^{-4} = 10^{-2}(2+3\cdot10^{-2}) = 10^{-2}(2+0{,}03) = 2{,}03\cdot10^{-2}$

$2\cdot10^{-2}+3\cdot10^{-6} = 10^{-2}(2+3\cdot10^{-4}) = 10^{-2}(2+0{,}0003) = 2{,}0003\cdot10^{-2} \approx 2\cdot10^{-2}$

Vežbanje:

1. $3\cdot10^{2}\cdot2\cdot10^{4}/2\cdot10^{3} = ?$
2. $2{,}5\cdot10^{2}\cdot4\cdot10^{3}/5\cdot10^{7} = ?$
3. $4\cdot10^{3}+2\cdot10^{5} = ?$
4. $2{,}5\cdot10^{2}+4\cdot10^{3} = ?$

Dekadni logaritmi

Primeri:

$100 = 10^2 \rightarrow 2 = \log 100$, $1000000 = 10^6 \rightarrow 6 = \log 1000000$

$5 = \log 10^5$, $7 = \log 10^7$, $12 = \log 10^{12}$

$-5 = \log 10^{-5}$, $-7 = \log 10^{-7}$, $-12 = \log 10^{-12}$

$\log 1 = 0$, $\log 10 = 1$

$10^2 \cdot 10^3 = 10^5 \rightarrow \log(a \cdot b) = \log a + \log b$

$10^8 / 10^3 = 10^5 \rightarrow \log(a/b) = \log a - \log b$

$(10^2)^4 = 10^8 \rightarrow \log (a^n) = n \cdot \log a$

$\sqrt{10} = 10^{1/2} \approx 3{,}16 \rightarrow \log 3{,}16 \approx 0{,}5$

$\log 2 \approx 0{,}30103$

$\log 4 = \log 2^2 = 2 \cdot \log 2 \approx 0{,}60206$

$\log 5 = \log(10/2) = \log 10 - \log 2 \approx 0{,}69097$

$\log 8 = \log 2^3 = 3 \cdot \log 2 \approx 0{,}90309$

$\log 200 = \log 2 \cdot 100 = \log 2 + \log 100 \approx 0{,}30103 + 2 = 2{,}30103$

$\log 0{,}04 = \log 4 \cdot 10^{-2} = \log 4 + \log 10^{-2} \approx 0{,}60206 - 2 = -1{,}39794$

$\log 2 \cdot 10^6 = \log 2 + \log 10^6 \approx 0{,}30103 + 6 = 6{,}30103$

$\log 4 \cdot 10^{-6} = \log 4 + \log 10^{-6} \approx 0{,}60206 - 6 = -5{,}39794$

$50 = 10 \cdot \log (I/10^{-12}) \rightarrow 5 = \log (I/10^{-12}) \rightarrow I/10^{-12} = 10^5 \rightarrow I = 10^{-12} \cdot 10^5 = 10^{-7}$

Vežbanje:

1. $\log 3 = 0{,}47712 \rightarrow \log 6 = ?$ $\log 9 = ?$
2. $\log 800 = ?$
3. $\log 900 = ?$
4. $\log 64 = ?$
5. $\log 96 = \log (3 \cdot 32) = ?$
6. $\log 2 \cdot 10^{14} = ?$
7. $\log 6 \cdot 10^{-13} = ?$
8. $\log x = 8 \rightarrow x = ?$
9. $\log x = -4 \rightarrow x = ?$

Linearne jednačine sa jednom nepoznatom

Primeri:

$2\cdot x = 8 \rightarrow x = 8/2$

$a\cdot x = b \rightarrow x = b/a$

$x/2 = 8 \rightarrow x = 2\cdot 8$

$\sigma = F/S \rightarrow F = \sigma\cdot S$

$8/x = 2 \rightarrow x = 8/2$

$U/R = I \rightarrow R = U/I$

$p\cdot Q\cdot x = m\cdot g \rightarrow x = m\cdot g/p\cdot Q$

$F\cdot x\cdot t/m\cdot v = Rb \rightarrow x = R\cdot b\cdot m\cdot v/F\cdot t$

$E/F\cdot x = pq^2 \rightarrow x = E/Fpq^2$

$2\cdot x+8 = 12 \rightarrow 2\cdot x = 12-8 \rightarrow x = (12-8)/2$

$p_0+mgx/S = p_A \rightarrow mgx/S = (p_A - p_0) \rightarrow x = S(p_A - p_0)/mg$

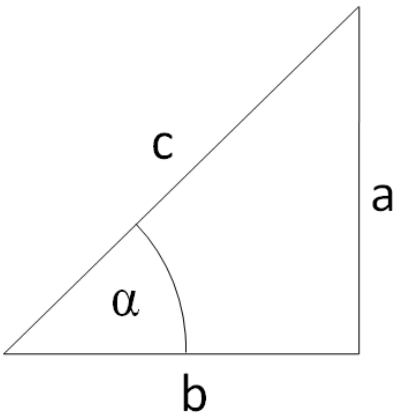
$4(3x-2) = 16 \rightarrow 3x-2 = 16/4 \rightarrow 3x = 16/4+2 \rightarrow x = (16/4+2)/3$

$$\frac{na}{h}\left(\frac{RT}{x}-\frac{mg}{Q}\right)=\frac{3p}{n}\rightarrow\frac{RT}{x}-\frac{mg}{Q}=\frac{3ph}{n^2a}\rightarrow\frac{RT}{x}=\frac{3ph}{n^2a}+\frac{mg}{Q}\rightarrow x=\frac{RT}{\frac{3ph}{n^2a}+\frac{mg}{Q}}$$

Vežba:

1. $p\cdot q\cdot x/m\cdot d = kr \rightarrow x = ?$
2. $3\cdot b\cdot x/c = ae^2-r \rightarrow x = ?$
3. $k\cdot m\cdot x/p-n = f\cdot r \rightarrow x = ?$
4. $p\cdot q/m\cdot x\cdot d = kr \rightarrow x = ?$
5. $3b/x\cdot c = ae^2-r \rightarrow x = ?$
6. $k\cdot m/p\cdot x-n = f\cdot r \rightarrow x = ?$

Trigonometrija



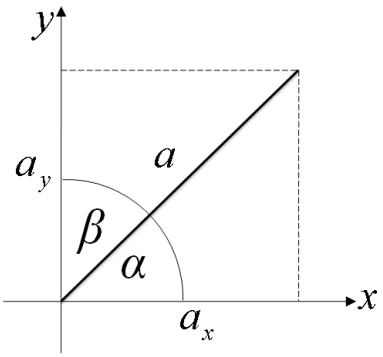
$\sin\alpha = a/c \rightarrow a = c\cdot\sin\alpha$

$\cos\alpha = b/c \rightarrow b = c\cdot\cos\alpha$

$\operatorname{tg}\alpha = a/b \rightarrow a = b\cdot\operatorname{tg}\alpha$

$\sin^2\alpha+\cos^2\alpha = 1$

$\sin\alpha / \cos\alpha = \operatorname{tg}\alpha$



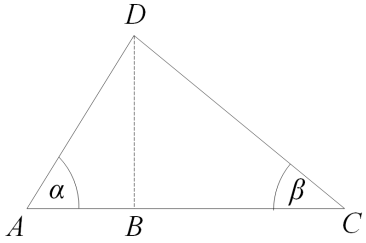
$a_x = a\cdot\cos\alpha \quad a_y = a\cdot\cos\beta$

α	sin α	cos α	tg α
0^0	$\sqrt{0}/2=0$	$\sqrt{4}/2=1$	0
30^0	$\sqrt{1}/2=1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45^0	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60^0	$\sqrt{3}/2$	$\sqrt{1}/2=1/2$	$\sqrt{3}$
90^0	$\sqrt{4}/2=1$	$\sqrt{0}/2=0$	∞

$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$

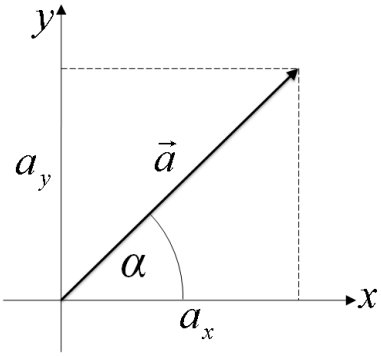
$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$

Vežba:



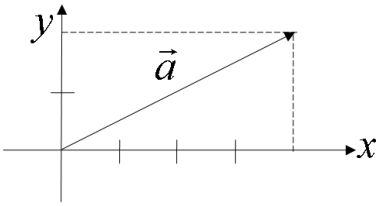
1. Ako je poznato da ugao α na slici iznosi 60^0, ugao β iznosi 30^0, a da je dužina AB jednaka 1 cm, odrediti redom dužine AD, BD, CD i BC, pa na kraju odrediti dužinu AC.

Vektori



$a_x = a \cdot \cos\alpha \quad a_y = a \cdot \sin\alpha$

$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$



$\vec{a} = (4,2) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Skalarni proizvod:

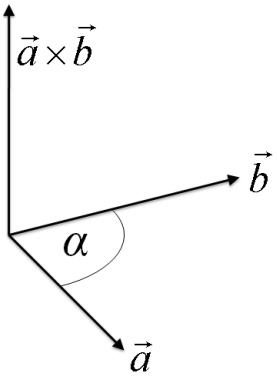
Proizvod je skalar – običan broj.

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\vec{a}, \vec{b}) = a \cdot b \cdot \cos\alpha$

Skalarni proizvod uzajamno normalnih vektora jednak je nuli

Vektorski proizvod:

Proizvod je vektor:



Intenzitet: $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin(\vec{a}, \vec{b}) = a \cdot b \cdot \sin\alpha$

Pravac: normalan na oba činioca

Smer: Pravilo desne ruke

Vektorski proizvod vektora istog pravca jednak je nuli

Rešenja

Razlomci

1. 3/4
2. 5/6
3. (ae+bdc)/bce=?
4. (x+2y)/xy
5. kg/s
6. $kg \cdot s^3/m^2$

Stepenovanje

1. a^9
2. a
3. 1/a
4. a^{2x+y}
5. a^{2x+3y}
6. b^2/a^2
7. a^2/b
8. a/bc^3
9. a^2b/c^3
10. $x = y^{4/3}$

Eksponencijalni zapis brojeva

1. $3 \cdot 10^3$
2. $2 \cdot 10^{-2}$
3. $2{,}04 \cdot 10^5$
4. $4{,}25 \cdot 10^3$

Dekadni logaritmi

1. $\log 6 = \log(2 \cdot 3) = \log 2 + \log 3 = 0{,}77815$, $\log 9 = \log 3^2 = 2 \log 3 = 0{,}95424$
2. $\log(8 \cdot 100) = 2{,}90309$
3. $\log(9 \cdot 100) = 2{,}95424$
4. $\log(8^2) = \log(2^6) = 6 \cdot \log 2 = 1{,}80618$
5. $\log 3 + \log 2^5 = 1{,}98227$
6. 14,30103
7. -12,22185
8. $x = 10^8$
9. $x = 10^{-4}$

Linearne jednačine sa jednom nepoznatom

1. x = krmd/pq
2. $x = (ae^2-r) \cdot c/3b$
3. $x = (fr+n) \cdot p/km$
4. x = pq/mdkr

5. $x = 3b/c(ae^2-r)$
6. $x = km/(fr+n)\cdot p$

Trigonometrija

1. $AD = AB/\cos\alpha = AB/\cos 60^0 = 2$ cm

   $BD = AD\cdot\sin\alpha = AD\cdot\sin 60^0 = \sqrt{3}$ cm

   $CD = BD/\sin\beta = BD/\sin 30^0 = 2\sqrt{3}$ cm

   $BC = CD\cdot\cos\beta = CD\cdot\cos 30^0 = 3$ cm

   $AC = AB+BC = 4$ cm